Relation between Kinetic Energy and Temperature of a gas.

The relationship between Kinetic Energy and Temperature can be derived from the Kinetic gas equation.

We know from Kinetic gas equation

$$PV = \frac{1}{3} mnc^2 \quad \text{--- (1)}$$

or $PV = \frac{2}{3} \times \frac{1}{2} mnc^2$

or, $PV = \frac{2}{3} \times \frac{1}{2} Mc^2$ --- (2) [$\because mn = M$, M = Mass of the gas]

$\because PV = RT$ (For 1 mole of gas) --- (3)

From eqn (2) and (3)

$RT = \frac{2}{3} \times \frac{1}{2} Mc^2$

$RT = \frac{2}{3} \times K.E.$ [$\because K.E = \frac{1}{2} Mc^2$] Where M is Mass & c is velocity

$\therefore K.E = \frac{3}{2} RT$

or $K.E \propto T$ [Where $\frac{3}{2}R$ is Constant]

So, It is clear that K.E of translation of an ideal gas is independent of the nature of the gas and its pressure. It depends only upon the temperature of the gas.

Derivation of the gas-laws on the basis of Kinetic gas equation.

(1) Derivation of the Boyle's Law:-

According to Kinetic theory of gases, the average Kinetic energy ($\frac{1}{2} mnc^2$) is directly proportional to absolute Temperature (T)

i.e. $\frac{1}{2} mnc^2 = RT$

$\frac{3}{2} \times \frac{1}{3} mnc^2 = RT$

$\frac{3}{2} P.V = RT$ [$\because \frac{1}{3} mnc^2 = PV$]

or $PV = \frac{2}{3} RT$

Therefore, the product of Pressure and volume is a constant at a constant temperature, i.e

$$PV = \text{Constant}$$

or $P \propto \frac{1}{V}$ at Constant Temperature T

This is Boyle's law

2. Derivation of Charle's Law from Kinetic gas Equation -

We know,

$$PV = \frac{2}{3} RT$$

or $V = \frac{2}{3 \cdot P} R.T$

$$V = \frac{2R}{3P} \cdot T$$ ——

If Pressure P is Constant,

then $V \propto T$

3. Avagadro's Law Derivation :-

When two gases have the same Pressure and Volume, then $P_1V_1 = P_2V_2$

We know from Kinetic gas eqn $PV = \frac{1}{3} mnc^2$

Applying Kinetic gas eqn

$$\frac{1}{3} m_1 n_1 c_1^2 = \frac{1}{3} m_2 n_2 c_2^2$$

$$\frac{2}{3} \cdot \frac{1}{2} m_1 n_1 c_1^2 = \frac{2}{3} \cdot \frac{1}{2} m_2 n_2 c_2^2$$

or $\frac{1}{2} m_1 n_1 c_1^2 = \frac{1}{2} m_2 n_2 c_2^2$ —— (1)

When the temperature of these two gases is also the same, then their Kinetic energy per mole will also be the same

i.e $\frac{1}{2} m_1 c_1^2 = \frac{1}{2} m_2 c_2^2$ —— (2)

Dividing equation (1) by (2)

we get, $n_1 = n_2$

Thus the equal volumes of all gases under the same Conditions of temperature and Pressure have the same number of moles.

This is Avagadro's Law.